Direct Numerical Simulation on the Receptivity, Instability, and Transition of Hypersonic Boundary Layers

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Abstract
The prediction of the laminar-turbulent transition of boundary layers is critically important to the development of hypersonic vehicles because the transition has a first-order impact on aerodynamic heating, drag, and vehicle operation. The success of transition prediction relies on a fundamental understanding of the relevant physical mechanisms. In the 20 years since the review by Kleiser & Zang (1991) on the direct numerical simulation (DNS) of the boundary-layer transition, significant progress has been made on DNS in the hypersonic flow regime and in the spatial DNS approach. Many high-order shock-capturing and shock-fitting finite-difference methods have been developed and extensively applied to numerical simulations of the hypersonic boundary-layer transition. DNS has become a powerful research tool and has led to discoveries of new transition mechanisms. This article reviews the recent progress of DNS on hypersonic boundary-layer receptivity, instability, and transition. The current status and future directions are also presented.
1. INTRODUCTION

The prediction of the laminar-turbulent transition of boundary layers is critically important to the development of hypersonic vehicles that are to be used for rapid space access (Kimmel 2003). The boundary-layer transition has a first-order impact on aerodynamic heating, drag, and vehicle operation because turbulent flows generate much higher friction and heating to the body than laminar ones. It also affects engine performance and operability, as well as the structure and weight of the vehicles. The success of transition prediction relies on a fundamental understanding of the relevant physical mechanisms leading to transition. Despite considerable efforts in experimental, theoretical, and numerical studies, many critical physical mechanisms underlying the hypersonic boundary-layer transition are still poorly understood (Morkovin 1987, Reshotko 2008).

In general, transition is a result of the nonlinear response of a laminar boundary layer to various environmental disturbances (Saric et al. 2002). Figure 1 schematically shows paths to the boundary-layer transition. In a weak-disturbance environment, the path to transition consists of three stages: receptivity, linear eigenmode growth, and nonlinear breakdown to turbulence. The receptivity process (Morkovin 1969) converts external forcing into boundary-layer waves and provides their initial conditions of amplitudes, frequencies, and phases. Modal growth of the unstable boundary-layer waves is linear and can be obtained by solving the eigenvalue problem of the homogeneous linearized stability equations. Breakdown to turbulence is caused mainly by the nonlinear secondary instabilities when the boundary-layer waves reach certain amplitudes. This three-stage-transition mechanism is represented by path A in the figure. As the disturbance amplitude increases, transient growth, arising from the nonorthogonal nature of Orr-Sommerfeld and Squire eigenfunctions, becomes important. Weak transient growth provides a higher initial amplitude for modal growth (path B), whereas strong transient growth can lead directly to

![Figure 1](image-url)  
**Figure 1**  
Paths to boundary-layer transition with respect to the disturbance amplitude. Path A, three-stage-transition mechanism in a weak-disturbance environment; path B, three-stage-transition mechanism including weak transient growth; paths C and D, transition mechanisms with strong transient growth; path E, bypass transition to very strong disturbance. Figure taken from Reshotko (2008) with permission of the American Institute of Aeronautics and Astronautics.
nonlinear instabilities or breakdown to turbulence (paths C and D). For hypersonic boundary layers, major relevant instability waves are the first and second modes (Mack 1984), Görtler instabilities over concave surfaces (Saric 1994), attachment-line instabilities (Sesterhenn & Friedrich 2006), and three-dimensional cross-flow instabilities (Reed & Saric 1989). The flow undergoes nonlinear breakdown to turbulence after the exponential growth of instability waves reaches certain magnitudes (Kachanov 1994).

Various physical mechanisms of boundary-layer transition have been reviewed by many authors over the years. Most of the previous reviews are limited to incompressible flows. An early review by Tani (1969) mainly focused on the incompressible boundary-layer transition in general and the nonlinear breakdown experiments by Klebanoff et al. (1962) in particular. Other reviews include Orszag & Kells (1980) on the transition of plane Poiseuille and Couette flows, Herbert (1988) on the secondary instabilities of boundary-layer flows, Goldstein & Hultgren (1989) on the receptivity of boundary layers to long-wave free-stream disturbances, Kachanov (1994) on the physical mechanisms of the boundary-layer transition with emphasis on nonlinear breakdown experiments, Saric (1994) on boundary-layer instabilities involving Görtler vortices, Saric et al. (2002, 2003) on advances in boundary-layer receptivity and cross-flow instability to external acoustic and vortical disturbances, Schmid (2007) on nonmodal stabilities of wall-bounded shear flows, and Durbin & Wu (2007) on theoretical and numerical research on the boundary-layer transition to vertical disturbances.

Compared to incompressible flows, transition mechanisms of hypersonic and supersonic boundary layers are much more complex and much less understood (Morkovin 1987; Reshotko 1991, 2008). Recently, Reed & Saric (1996) reviewed the use of linear stability theory (LST) for transition predictions of compressible boundary layers. Schneider wrote a series of reviews on available flight data and wind tunnel experiments of the hypersonic boundary-layer transition for circular cones, scramjet forebodies, and reentry capsules (Schneider 1999, 2004, 2006); the effects of high-speed tunnel noise on transition (Schneider 2001); and experimental and flight tests of the hypersonic boundary-layer transition with surface roughness (Schneider 2008a,b). More recently, Fedorov (2011) summarized theoretical models and the analysis of hypersonic boundary-layer stability and transition, with emphasis on qualitative features of the disturbance spectrum leading to new mechanisms of receptivity and instability.

Most of our current knowledge on hypersonic boundary-layer stability and transition is based on LST (Herbert 1997, Mack 1984), which concerns the exponential growth or decay of boundary-layer waves. Mack (1984) found that there are higher acoustic instability waves in addition to the first mode in supersonic and hypersonic boundary layers. Among them, the second mode becomes the dominant instability at Mach numbers larger than approximately 4. The transition of hypersonic boundary layers over blunt bodies is affected by the additional effects of shock waves (Reshotko 1991), entropy-layer instabilities (Stetson et al. 1984), nose bluntness, and thermochemical nonequilibrium at high temperatures (Johnson et al. 1998, Ma & Zhong 2004). Figure 2 shows a schematic of the wave field in a hypersonic flow induced by freestream disturbance and surface roughness.

Because of advances in numerical methods and computer technologies, direct numerical simulation (DNS) has become a powerful tool in the study of transitional (Kleiser & Zang 1991), as well as turbulent (Moin & Mahesh 1998), boundary-layer flows. In DNS studies, the boundary-layer transition is simulated by numerically solving the three-dimensional Navier-Stokes equations. It is necessary to use highly accurate numerical methods to resolve a wide range of timescales and length scales in the flow. DNS is not only a useful research tool for understanding and discovering hypersonic boundary-layer transition mechanisms, but also a practical tool in the evaluation of transition-prediction methods. Kleiser & Zang (1991) reviewed DNS studies on the transition of
Figure 2
A schematic of the wave field in a hypersonic flow induced by free-stream disturbance and surface roughness. Figure taken with permission from Zhong (1998).

For the boundary-layer transition, there are two types of DNS studies: temporal DNS (TDNS) and spatial DNS (SDNS) (Kleiser & Zang 1991). In TDNS, it is assumed that disturbances are periodic in the streamwise direction. Therefore, TDNS can simulate the temporal development of disturbances within a very small domain in the streamwise direction, which significantly lowers the requirements for computer time and memory. However, TDNS neglects the effects of spatial growth of the boundary layer. Guo et al. (1995) modified the TDNS method so that nonparallel effects could be included. Alternatively, SDNS is conducted within a much larger domain in the streamwise direction so that the spatial development of the boundary layer and disturbances can be investigated. Although SDNS requires much more computer resources and more efficient numerical algorithms, it is preferable to TDNS in transition research. At the time of Kleiser & Zang’s review, most of the DNS work was limited to TDNS of incompressible boundary layers. The more realistic SDNS on the boundary-layer transition was still in an early stage.

In the 20 years since Kleiser & Zang’s review, significant progress has been made on DNS in the hypersonic flow regime and in the SDNS approach. With advances in computer power and efficient parallelizable numerical algorithms, DNS has become a powerful and mature tool for research on the hypersonic boundary-layer transition and has led to new discoveries in transition mechanisms. Some of the progress in the DNS of the hypersonic boundary-layer transition has been reviewed by Rempfer (2003), Fasel (2006), and Reed (2008). Rempfer and Reed mainly focused on incompressible flow with a brief mention of the high-speed boundary-layer transition. Fasel’s (2006) short review concentrated on the DNS of nonlinear breakdown for both low- and high-speed boundary-layer flows. The objective of this article is to review the recent progress of the DNS on hypersonic boundary-layer receptivity, instability, and transition since 1991. We also present the current status and future directions of this area. Although the main focus is on the hypersonic boundary-layer transition, supersonic boundary-layer flows are also covered as the flow physics in these two regimes is closely related.
2. MATHEMATICAL MODELS AND GOVERNING EQUATIONS

For the DNS of hypersonic boundary-layer stability and transition, the flow is mainly in the continuum regime in which the Navier-Stokes equations are valid. With few exceptions, past and current DNS studies have been mainly based on the perfect gas model, which is appropriate for cold or low-enthalpy hypersonic flows in most wind tunnels. In general, the unsteady three-dimensional Navier-Stokes equations are written in the following conservation-law form:

$$\frac{\partial U}{\partial t} + \frac{\partial F_j}{\partial x_j} + \frac{\partial F_{vj}}{\partial x_j} = W,$$  \hspace{1cm} (1)

where $U = (\rho u, \rho u_2, \rho u_3, \rho u, \rho u + p)$, and

$$F_j = \begin{cases} 
\rho u_j \\
\rho u_1 u_j + p \delta_{1j} \\
\rho u_2 u_j + p \delta_{2j} \\
\rho u_3 u_j + p \delta_{3j} \\
(e + p)u_j 
\end{cases}, \hspace{1cm} (2)$$

$$F_{ij} = \begin{cases} 
0 \\
\tau_{ij} \\
\tau_{2ij} \\
\tau_{3ij} \\
\tau_{ijkl} u_k - q_j
\end{cases}, \hspace{1cm} (3)$$

In Equation 1, $W$ is the source term introduced by thermochemical nonequilibrium processes. For a perfect gas, $\rho_m$ is the same as the total density $\rho$, and the source term is zero. For a nonequilibrium real gas, $\rho_m$ represents densities of individual species, and the specific formulas for the source term are determined by specific nonequilibrium models used in DNS. The transport equations are

$$\tau_{ij} = \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} \mu \frac{\partial u_k}{\partial x_k} \delta_{ij}, \hspace{1cm} (4)$$

$$q_j = -\kappa \frac{\partial T}{\partial x_j}, \hspace{1cm} (5)$$

where $\mu$ and $\kappa$ are the gas viscosity and heat conductivity, respectively.

For hypersonic flows with high stagnation enthalpy (hypervelocity flow), which is the case for hypersonic vehicles, real gas effects become too significant to be neglected. For airflow in the boundary layer around vehicles flying in the atmosphere, these effects include excitation of the vibration energy, the dissociation and recombination of gas species, the ionization of gas species, and radiation (Bertin & Cummings 2006). In addition, there may be effects of surface catalytic reactions and the ablation of thermal protection materials on the surface. Significant progress has been made on the development of nonequilibrium models for hypersonic flow simulations (Gnoffo et al. 1989, Gupta et al. 1990, Stalker 1989). LAURA, DPLR, and US3D are the most frequently referenced nonequilibrium flow solvers (Gnoffo et al. 1989, Nompelis et al. 2005, Wright et al. 1998) based on these models and are intensively validated (Hash et al. 2007). Recently, a high-order shock-fitting nonequilibrium flow solver has been developed and validated by Wang & Zhong (2011a). The new solver is based on the generally used 11-species chemistry set of air consisting of $N_2$, $O_2$, $NO$, $N$, $O$, $N_2^+$, $O_2^+$, $NO^+$, $N^+$, $O^+$, and $e$ and a two-temperature model. It is assumed that translation and rotation energy modes are in equilibrium at the translation temperature, whereas the vibration energy, electronic energy, and free-electron energy are in equilibrium at the vibration temperature. The source terms in Equation 1 are determined by the models of the vibration and
3. NUMERICAL METHODS

Most DNS studies on hypersonic boundary-layer transition are carried out for relatively simple geometries, such as flat plates, sharp or blunt wedges, and sharp or blunt cones. Flow fields over these objects can be modeled by smooth structured grids, in which governing equations (Equation 1) in the Cartesian coordinate system are transformed from the body-fitted physical domain to the rectangular computational domain in a curvilinear coordinate system. Figure 3 shows an example of the body-fitted grid for hypersonic flow over a blunt body bounded by the bow shock and the body surface. Highly accurate finite-difference or finite-volume schemes are subsequently applied to solve the transformed equations. Because transition is highly dependent on the initial and operating conditions, Reed et al. (1998) concluded that finding careful, archival experiments for comparison is the main validation issue for computational fluid dynamics modeling of transition.

3.1. Shock Capturing Versus Shock Fitting

Currently, two main approaches have been used for the DNS of the hypersonic boundary-layer transition. The first approach is to use high-order, low-dissipative, and nonlinear shock-capturing
schemes (Pirozzoli 2011). For example, Balakumar and colleagues (Balakumar 2009, Balakumar & Kegerise 2010) used a fifth-order weighted essentially non-oscillatory (WENO) scheme (Liu et al. 1994) to study the receptivity and instability of hypersonic boundary layers. Other high-order shock-capturing schemes include hybrid WENO/central-difference schemes, schemes with artificial diffusivity, and adaptive characteristic-based filter schemes that can capture shock waves by introducing local numerical dissipation (Johnsen et al. 2010). Shock-capturing schemes are extremely robust in the presence of strong shock waves and complex shock interactions. They can simulate hypersonic boundary-layer transition problems, including the interaction of shock and very strong disturbances. In addition, implementation of computer code for shock-capturing schemes is straightforward. The main disadvantage of shock-capturing schemes is that they reduce to first-order accuracy at the shock, and there may be spurious oscillations behind strong shocks (Rawat & Zhong 2010). Another disadvantage is that shock-capturing schemes are computationally expensive relative to conventional finite-difference schemes because of the automated determination of high gradient zones.

The second approach is to treat the shock and its interaction with disturbances by high-order shock-fitting methods (Moretti 2002, Zhong 1998). By treating shocks as sharp interfaces, shock-fitting algorithms can achieve uniform high-order accuracy in the flow field (Rawat & Zhong 2010). They can also avoid spurious oscillations near strong shocks. In this approach, the motion of the shock is determined by flow conditions on both sides of the shock. The shock-fitting methods are uniquely suited for the DNS of hypersonic flows involving a single well-defined bow shock (Zhong 1998). However, the shock-fitting methods are limited to resolving only simple shock structures, for example, shocks over a blunt body. They fail to resolve the leading edge singularity and the hypersonic boundary-layer transition problems where the shock front is broken by very strong disturbances. The shock-fitting scheme is also computationally expensive relative to the conventional finite-difference scheme because of the special treatment of shock waves.

Therefore, for flow with complex shock interactions, shock-capturing schemes are preferred, whereas shock-fitting methods are better suited for flow with a well-defined shock. Sometimes, a combination of shock-capturing schemes and shock-fitting methods is more powerful for hypersonic boundary-layer transition problems. For example, for DNS of hypersonic boundary layers over a flat plate, wedge, or sharp cone, shock-capturing methods are generally used to solve the flow field near the leading edge, whereas shock-fitting methods are used for flow field further downstream (Wang & Zhong 2011b). Because high-order shock-capturing methods that are suitable for the DNS of turbulent, as well as transitional, compressible boundary-layer flows in the presence of shock waves were reviewed by Pirozzoli (2011), we do not discuss the details of these methods here. Instead, we discuss high-order shock-fitting schemes and linear high-order schemes for the DNS of hypersonic boundary layers in the following sections. Finally, applications of shock-capturing and shock-fitting methods on hypersonic boundary-layer transitions are reviewed.

3.2. High-Order Shock-Fitting Schemes

Shock-capturing schemes are locally only first-order accurate at the shock and may incur spurious numerical oscillations near a strong shock (Rawat & Zhong 2010). Shock-fitting algorithms have been proposed as an alternative that can achieve uniform high-order accuracy in the flow field and avoid possible spurious oscillations near the shock. Shock-fitting schemes have been used for simulations of compressible flow with well-defined shocks since the 1960s (Moretti 1987). The shock-fitting method has also been applied to hypersonic flows together with a spectral method (Hussaini et al. 1985).
Zhong (1998) developed a high-order shock-fitting method for the DNS of three-dimensional hypersonic boundary layers with an unsteady bow shock. The flow variables behind the shock are determined by a combination of the Rankine-Hugoniot relations across the shock and a characteristic compatibility equation from behind the shock. There is no numerical smearing of the shock front so that it can be represented by a shock height function \( H = H(\xi, \tau) \), which is the distance along a wall-normal grid line from the wall surface to the shock, as schematically shown in Figure 3. The function \( H \) is governed by two additional equations of the shock velocity and acceleration in the following forms:

\[
\begin{aligned}
\frac{\partial H}{\partial \tau} &= H_t, \\
\frac{\partial H_t}{\partial \tau} &= H_{tt} \left( \xi, U_t, \frac{\partial U_t}{\partial \tau}, H, H_t, U_\infty, \frac{\partial U_\infty}{\partial \tau} \right),
\end{aligned}
\]

(6)

where \( H_{tt} \) is the acceleration of the shock front, and the subscripts \( s \) and \( \infty \) represent flow variables on the high-pressure and free-stream sides of the shock, respectively. The two equations above are integrated in time simultaneously by using the same Runge-Kutta methods as those used for governing equations. Uniform high-order accuracy can be achieved for the entire flow field, including the shock, when all equations are spatially resolved by high-order finite-difference methods.

For flows involving secondary shocks behind the main bow shock, such as the case of flow over surface roughness of a finite height, a combined shock-fitting and shock-capturing approach can be used in which the main shock is fitted while the flow behind the main shock is computed using a high-order shock-capturing scheme. Rawat & Zhong (2010) performed a systematic assessment of the accuracy of the shock-fitting and shock-capturing methods for shock-disturbance interaction problems. They showed that a fifth-order convergence is indeed achieved by Zhong’s (1998) fifth-order shock-fitting scheme. More importantly, the computational errors of the pure fifth-order WENO scheme (WENO5) are approximately five orders of magnitude larger than those of the fifth-order shock-fitting scheme. The overall order of accuracy for WENO5 is reduced to first order because of the shock. For a two-dimensional problem of a Mach-1.5 shock interacting with weak planar vorticity-entropy waves, Figure 4 compares density distributions obtained from WENO5 and those computed by two different versions of shock-fitting schemes. It is observed that, at an incident angle of 75°, WENO5 produces spurious oscillations behind the shock, which reduces the scheme to first-order accuracy near the shock. However, such spurious oscillations do not exist in shock-fitting solutions.

### 3.3. High-Order Finite-Difference Methods

Traditional numerical methods for the DNS of transitional and turbulent flows are spectral methods because of their high accuracy. However, the applications of spectral methods are mainly limited to flows in simple domains. Finite-difference methods, including explicit and compact schemes, have received much attention for the DNS of hypersonic boundary-layer transition because they can be easily applied to complex geometries (Lele 1992, Pruett et al. 1995, Rai & Moin 1993, Zhong 1998). Most high-order finite-difference methods are central schemes, which introduce only phase errors but no dissipative errors (Lele 1992, Pruett et al. 1995). However, central schemes are not robust enough for simulations of convection-dominated hypersonic flow. Extra filtering procedures, which are equivalent to adding numerical dissipation, are needed to stabilize central schemes and control the aliasing errors.
Shock-fitting with upwind scheme
Shock-fitting with WENO5
WENO5 scheme in entire domain

1.91

0

1

2

3

4

5

6

7

x

1.90

1.89

1.88

1.87

y

ρ

a

b

Figure 4
Density distributions obtained from WENO5 and two shock-fitting schemes: (a) instantaneous contours and (b) profiles along the $y = \pi$ line. Figure taken with permission from Rawat & Zhong (2010).

Many researchers (Adams & Kleiser 1996, Pruett et al. 1995) have used high-order compact schemes optimized for spectral-like resolution based on the idea of Lele (1992). Visbal & Gaitonde (2002) developed very-high-order finite-difference schemes for the simulation of compressible flows on stretched, curvilinear, and deforming meshes. Their sixth-order compact spatial discretizations are coupled with up to tenth-order low-pass filters. Rizzetta & Visbal (2006) used these schemes for the DNS of compressible flow over an array of distributed roughness elements. Alternatively, upwind-bias high-order schemes are very robust for compressible flow simulations (Rai & Moin 1993, Zhong 1998). The inherent numerical dissipation in these schemes is enough to control the aliasing errors. Recently, Subbareddy & Candler (2009) presented a fully discrete, kinetic-energy consistent finite-volume scheme, which is a low-dissipative and robust method. A shock is captured by switching on a dissipative flux term, which tends to zero in smooth regions of the flow. This method was incorporated in the simulation by Bartkowicz et al. (2010a) for roughness-induced instability in a Mach-6 wind tunnel.

Zhong (1998) presented a family of grid–centered upwind compact and explicit finite-difference schemes of third, fifth, and seventh order and their stable high-order boundary schemes for the DNS of the hypersonic boundary-layer transition. These upwind schemes are derived on central grid stencils so that dissipative errors are smaller than the dispersive errors inherent in equivalent central schemes and are large enough to stabilize high-order inner schemes coupled with boundary closure schemes. The accuracy of the upwind schemes is one order lower than the maximum order the central stencils can achieve so that there is an adjustable coefficient in the leading dissipative truncation term. Specifically, a fifth-order finite-difference formula on a seven-point uniform-grid stencil for the first-order derivative of inviscid flux can be written as

$$
\left( \frac{\partial F}{\partial \xi} \right)_i = \frac{1}{b} \sum_{k=-3}^{3} a_{i+k} F_{i+k} - \frac{\alpha}{6!} \left( \frac{\partial^6 F}{\partial \xi^6} \right)_i + \ldots,
$$

where $b$ is the grid size. The coefficients are calculated using a Taylor expansion: $a_i = \frac{1}{20} (-\frac{1}{4} \alpha)$, $a_{i+1} = \frac{1}{60} (-45 + \frac{5}{4} \alpha)$, $a_{i+2} = \frac{1}{60} (19 - \frac{1}{4} \alpha)$, and $a_{i+3} = \frac{1}{60} (\pm 1 + \frac{1}{12} \alpha)$. The specific value of $\alpha$
is chosen based on a stability analysis. For \( \alpha < 0 \), Equation 7 represents a fifth-order upwind scheme. When \( \alpha = 0 \), the scheme is a sixth-order central scheme used for the viscous terms. The recommended value for \( \alpha \) is \(-6\), which is less dissipative than a forward upwind scheme. The high-order upwind schemes can be applied to the Euler equations by means of the Lax-Friedrichs method.

The fifth-order grid-centered upwind scheme has been extensively applied by Zhong’s group at UCLA (Zhong & Ma 2006) to DNS studies of hypersonic boundary-layer receptivity and stability and strong shock-turbulence interactions (Rawat & Zhong 2010). Recently, grid-centered upwind schemes of up to ninth order were used by Sivasubramanian & Fasel (2010) for the nonlinear breakdown of supersonic and hypersonic boundary layers.

3.4. High-Order Cut-Cell Methods

The hypersonic boundary-layer transition can be significantly affected by the existence of surface roughness (Reda 2002, Schneider 2008a). Owing to the complex geometry of the embedded roughness, it is difficult to generate body-fitted structured grids for three-dimensional flow fields with roughness. The use of unstructured grids makes it possible to treat arbitrary roughness, but it is a challenge to use high-order finite-difference methods on unstructured grids. One approach to overcome the difficulty in grid generation and the challenge in using high-order finite-difference methods is to use a Cartesian grid method in which the grid lines are not aligned with the body surface. Various Cartesian grid methods have been developed and used to solve problems with arbitrary geometries (Peskin 2002, Mittal & Iaccarino 2005).

Peskin (2002) developed the immersed boundary method for the simulation of blood flow in hearts in which the surface of a solid membrane is represented by a discrete delta function. The delta function is added into the Navier-Stokes equations, and the resulting equations are discretized by a standard finite-difference method. This method has been applied to many problems, including flow interactions with solid surfaces (Marxen & Iaccarino 2008). The discrete-delta-function approach leads to a smeared interface with a thickness on the order of a mesh width. As a result, the immersed boundary method is locally first-order accurate at the interface.

Recently, Duan et al. (2010) developed and tested a new high-order cut-cell method for the DNS of the hypersonic boundary-layer transition with surface roughness. This method combines a nonuniform-grid finite-difference method for grid points near the roughness element and a shock-fitting method for the bow shock. For two- and three-dimensional problems, the cut-cell method is implemented in a dimension-by-dimension manner, i.e., the flux terms in wall-normal and spanwise directions are treated similar to those in the streamwise direction. The new cut-cell method has been successfully applied to a three-dimensional Mach-5.92 flow over a flat plate with an array of surface-roughness elements (Duan & Zhong 2010). The effects of the location of the roughness elements on the instability of the boundary layer have been studied in numerical simulations. Park & Mahesh (2007) developed a new algorithm for a compressible-flow simulation on unstructured grids. A least-squares approach is used to reconstruct the fluxes at cell interfaces. Shock waves are captured by a characteristic-based filter algorithm. This algorithm was used by Iyer et al. (2011) for DNS of the hypersonic boundary-layer transition induced by surface roughness.

3.5. Stable High-Order Boundary Closure Schemes

The DNS of the receptivity, instability, and transition of hypersonic boundary layers requires high-order schemes as lower-order schemes cannot resolve well the wide range of timescales and length scales in such flows. One of the main limiting factors for the use of very-high-order schemes
is the instability of high-order boundary closure schemes (Zhong 1998). Carpenter et al. (1993) showed that for a sixth-order compact central scheme, only a third-order boundary scheme can be used without introducing instability. Zhong (1998) showed that his fifth-order grid-centered upwind scheme coupled with a fourth-order boundary closure scheme is asymptotically stable. Therefore, globally fifth-order accuracy is maintained. Adams & Kleiser (1996) used a sixth-order compact scheme in the wall-normal direction and third- to fourth-order boundary closures for the first- and second-derivative operators.

Because stable and very-high-order boundary closures for a uniform grid are difficult to obtain, Zhong & Tatineni (2003) proposed a family of very-high-order nonuniform-grid finite-difference schemes with stable boundary closures. The order of these schemes ranges from first at the lowest to the global spectral collocation method at the highest. With the new schemes, the instability of boundary closures is overcome for arbitrarily high-order finite-difference schemes in the interior and at the boundaries. The schemes are derived directly on a nonuniform stretched grid without coordinate transformation to a uniform grid. The nonuniform grid is given by the following stretching function:

$$x_j = \sin^{-1}(-\alpha \cos(\pi j / N)) / \sin^{-1} \alpha,$$

where \(j\) is the grid index and \(N\) is the number of grid points. The parameter \(\alpha\) is a positive parameter used to change the stretching of the grid from one limit of a Chebyshev grid at \(\alpha \rightarrow 0\) to the other limit of a uniform grid at \(\alpha = 1\).

The high-order nonuniform-grid schemes (up to eleventh order) were subsequently applied to the receptivity simulation of a hypersonic boundary layer over a blunt leading edge to free-stream disturbances (Zhong & Tatineni 2003). The results showed that these schemes are stable and are able to produce high-accuracy solutions. The new methods were extended by Shukla & Zhong (2005) to a family of very-high-order nonuniform-grid compact finite-difference schemes with spatial order of accuracy ranging from fourth to twentieth. Recently, the nonuniform-grid high-order schemes were used by Sivasubramanian & Fasel (2010) to obtain stable boundary closures for their ninth-order schemes in the DNS of the hypersonic boundary-layer transition.

3.6. Time-Advancement Schemes

In most DNS calculations of the hypersonic boundary-layer transition for a perfect gas, explicit third- or fourth-order Runge-Kutta schemes are used to integrate the Navier-Stokes equations in time (Adams & Kleiser 1996, Pruett et al. 1995). However, the perfect gas assumption becomes unrealistic for hypersonic flows with high enthalpy because flow temperatures increase significantly across the bow shock. The gas between the shock and body surface becomes thermally excited and chemically reactive. The source terms introduced to the governing equations by real gas effects are often stiff for temporal integration. Zhong (1996) developed a set of third-order semi-implicit Runge-Kutta schemes for high-order temporal integration of the governing equations with stiff thermochemical source terms. These schemes are able to compute nonequilibrium flows with third-order temporal accuracy and are unconditionally stable.

4. BOUNDARY CONDITIONS

For simulations of hypersonic boundary-layer stability and transition, boundary conditions on the computational domain are important for an accurate solution (Bodony 2006, Kloker & Konzelmann 1993, Poinset & Lele 1992) and for the stability of high-order numerical
algorithms (Zhong & Tatineni 2003). The wall can be either isothermal or adiabatic for a steady base flow. The physical boundary condition of velocity on the wall is a no-slip condition. High-order extrapolation is generally used for outlet conditions because the flow is hypersonic at the exit, except for a small region near the wall. If the waves in the flow are nonlinear, a buffer (sponge) zone is often needed at the exit to damp out spurious wave reflections (Bodony 2006, Kloker & Konzelmann 1993). One can also use characteristic boundary conditions at the exit of the computational domain (Poinset & Lele 1992). For shock-capturing methods, the inlet and upper boundary conditions are the same as the free-stream conditions. For shock-fitting methods, flow variables behind the shock, as shown in Figure 2, are solved by combining Rankine-Hugoniot relations across the shock and a characteristic compatibility relation coming from the downstream flow (Zhong 1998). In stability and transition simulations, forcing waves are often introduced at or near the inlet to induce instability waves. Examples of forcing waves include periodic wall disturbances, free-stream disturbances, surface roughness, or boundary-layer waves obtained by LST or PSEs. The temperature perturbations with respect to the base flow are often set to zero, which is the standard boundary condition for theoretical and numerical studies of high-frequency disturbances. If free-stream disturbances are considered, they are generally imposed at the inlet for shock-capturing methods or on the bow shock for shock-fitting methods (Zhong 1998).

5. RECEPTIVITY AND INSTABILITY

Figure 1 demonstrates that the transition of hypersonic boundary layers generally consists of receptivity, instability, and breakdown. In DNS studies, the receptivity and instability of boundary-layer waves are usually combined. This section presents DNS studies of the hypersonic boundary-layer receptivity and instability to free-stream disturbances, wall disturbances, and surface roughness as well as the real gas effects of high-enthalphy flow on the receptivity and stability. The main receptivity mechanisms of hypersonic boundary layers are different from those of low-speed boundary layers. Resonant interactions between forcing waves and boundary-layer waves are the main receptivity mechanisms in hypersonic boundary layers. In addition, real gas effects are found to stabilize boundary-layer flows.

5.1 Free-Stream Disturbances

Free-stream disturbances, including acoustic waves, turbulence, and entropy waves, are important disturbances that hypersonic vehicles experience in real flight conditions. Physically, acoustic waves are related to pressure disturbances, turbulence is related to vorticity disturbances, and entropy waves are related to temperature or density disturbances. Acoustics waves propagate at the speed of sound with respect to the flow, whereas turbulence and entropy waves travel with the flow. The receptivity and stability of hypersonic boundary layers to free-stream disturbances have been studied theoretically by Fedorov and colleagues since the 1990s (Fedorov 2011, Fedorov & Khokhlov 2001). The theoretical results agreed with the DNS results of Zhong (2001), Ma & Zhong (2003a,b, 2005), Malik & Balakumar (2005), Balakumar (2009), Balakumar & Kegerise (2010), Egorov et al. (2006b), and other researchers.

Zhong (2001) studied the leading-edge receptivity to free-stream disturbances for a two-dimensional Mach-15 flow over a parabola. The results indicated that the generation of boundary-layer waves mainly results from acoustic waves behind the bow shock, rather than entropy and vorticity waves. Ma & Zhong (2003a,b; 2005) conducted a series of studies on the receptivity of a Mach-4.5 flow over a flat plate to free-stream disturbances using DNS and LST. Figure 5 shows the phase velocities of the first several boundary-layer waves for two cases: at a fixed frequency with changing streamwise location and at a fixed streamwise location with changing
frequency. In the figure, there is a class of waves with the initial phase velocity of fast acoustic waves. After these waves appear, their phase velocities decrease gradually as they propagate downstream. These waves are defined by Ma & Zhong as mode I, II, III, etc., according to the sequence of their appearance. They are in fact the multiple-viscous solutions defined by Mack (1984) and mode F defined by Fedorov & Tumin (2010). At a lower frequency, the wave exchange between mode I and the Mack mode is observed, which was first found by Fedorov & Khokhlov (2001).

Ma & Zhong (2003a,b, 2005) also showed that when free-stream fast acoustic waves at zero incident angle are introduced near the leading edge, a number of boundary-layer waves are generated. Mode I is generated first because its phase velocity near the leading edge is very close to that of fast acoustic waves. Although mode I is predicted to be stable by LST, its amplitude initially grows because of the resonance with fast acoustic waves. After reaching a peak value, mode I decays owing to its stable property when it propagates further downstream. Before mode I dies out, it converts to the first mode at the synchronization point between mode I and the Mack mode. The results indicated that although mode I is always stable, it can play an important role in the receptivity process because it can convert to the unstable Mack mode, which is shown in Figure 6. For free-stream slow acoustic waves, the first mode is directly generated. After passing the synchronization point, the first mode becomes the second Mack mode. Furthermore, the second mode is qualitatively stronger than that for the case of free-stream fast acoustic waves. For free-stream entropy and vorticity waves, it was found that boundary-layer waves are mainly induced by fast acoustic waves generated behind the shock. As a result, the receptivity to entropy and vorticity waves is quite similar to that to fast acoustic waves.
Balakumar (2009) performed extensive DNS studies on the receptivity of hypersonic boundary layers over a blunted flat plate, blunt cones with nose-bluntness effects and wall-temperature effects (Kara et al. 2007, 2008), and straight and flared cones (Balakumar 2008, Balakumar & Kegerise 2010). For the receptivity of a three-dimensional Mach-3.5 flow over a blunted flat plate to fast acoustic disturbances, Balakumar (2009) conducted DNS with and without a two-dimensional isolated roughness element located near the leading edge. The simulations showed that linear instability waves are generated very close to the leading edge. The small isolated roughness element does not enhance the receptivity for the given nose bluntness. Similar to Ma & Zhong (2005), he also found that slow acoustic waves induce much stronger instability waves than fast acoustic waves. Egorov et al. (2006b) conducted simulations on the receptivity of a Mach-6 flat-plate boundary layer to acoustic disturbances by using a second-order total variation diminishing scheme. They showed that their numerical results for fast and slow acoustic waves agree qualitatively with asymptotic theory. Owing to the viscous-inviscid interaction, the shock formed near the leading edge may significantly affect the acoustic field and the receptivity.

Zhong & Ma (2006) carried out a DNS study of the receptivity to free-stream fast acoustic waves for a Mach-7.99 flow over a 7° half-angle blunt cone. It was found that neither the first mode nor the second mode is excited by fast acoustic waves in the early region along the cone surface, although the Mack mode is unstable there. Instead, the second mode is excited downstream of the second-mode branch I neutral point. The delay of the second-mode excitation results from the receptivity being governed by a two-step resonant process: (a) resonance between forcing waves and the stable mode I near the leading edge and (b) resonance between the induced mode I and the unstable Mack mode downstream.
5.2 Wall Disturbances

The receptivity and instability of hypersonic boundary layers to wall disturbances, including blowing suction, is also important. Wall disturbances, together with freestream disturbances, are the main disturbances that hypersonic vehicles experience under real flight conditions. Blowing suction is not only the most sensitive wall disturbance for hypersonic boundary layers but also widely used to control the boundary-layer transition. For example, Egorov et al. (2008) studied the effect of porous coating on the stability and receptivity of a Mach 6 flat-plate boundary layer. In their numerical simulations, a porous coating was modeled by pressure-perturbation-related wall blowing suction.

By solving the compressible linearized Navier-Stokes equations, Malik et al. (1999) investigated the responses of a Mach-8 flow over a sharp wedge to three types of external forcing: a planar free-stream acoustic wave, a narrow acoustic beam enforced on the bow shock near the leading edge, and a blowing/suction slot on the wedge surface. They concluded that these three types of forcing eventually result in the same type of instability waves in the boundary layer. Fedorov & Khokhlov (2002) studied the receptivity of a hypersonic boundary layer over a flat plate to wall disturbances using a combination of an asymptotic method and numerical calculation. They found that strong excitations occur in local regions in which forcing disturbances are resonant with the boundary-layer waves. Their theoretical results also showed that the receptivity of the hypersonic boundary layer to wall blowing and suction is much stronger than that to wall vibrations and temperature disturbances. Egorov et al. (2006a) studied unsteady two-dimensional flow relevant to the stability of a Mach-6 flat-plate boundary layer. For small-amplitude blowing/suction, the growth rate of Mack’s second mode computed by DNS agreed well with those predicted by the LST including nonparallel flow effects.

Wang et al. (2011c) studied the response of a Mach-8 flow over a sharp wedge to wall blowing and suction in which disturbances are introduced through an actuator on the wedge surface. Figure 7 shows the pressure perturbations along the wedge surface at the same frequency for the seven cases of simulations. The actuator is upstream of the corresponding synchronization point in cases 1 to 6, whereas it is downstream of the synchronization point in case 7. The figure shows that mode S is strongly excited in cases 1 to 6, as demonstrated by the significant increase in the pressure perturbation amplitude at approximately 0.7 m downstream of the leading edge. In the figure, mode S is defined by Fedorov & Tumin (2010) and is the Mack mode defined by Mack (1984). In case 7, there is no significant amplification of the pressure perturbation downstream of the actuator despite the fact that the actuator is still located within the unstable region of the second mode. Such a result is held for wall blowing and suction at all other frequencies considered.

For a Mach-5.92 boundary layer over a flat plate, Figure 8 shows the contour of pressure fluctuations induced by a streamwise velocity perturbation (Wang & Zhong 2009). Downstream of the forcing region, pressure fluctuations are divided into two branches. One branch radiates into the external flow outside the boundary layer and propagates along Mach lines (transmitted acoustic waves), whereas the other branch stays within the boundary layer (boundary-layer waves). The DNS results show that instability waves excited by wall blowing and suction have higher amplitudes than those excited by streamwise velocity and temperature perturbations, which is consistent with the theoretical results of Fedorov & Khokhlov (2002).

5.3 Surface Roughness

Practical hypersonic vehicles contain various kinds of rough surfaces, which include distributed or isolated roughness. Surface roughness can alter boundary-layer-instability characteristics
substantially and trigger transition within boundary layers. The effects of surface roughness on transition have been reviewed by Reda (2002) and Schneider (2008a,b). Reshotko & Tumin (2004) proposed a transient-growth model for transition due to distributed roughness using their spatial transient growth theory, in which they assumed a linear wave growth with respect to the roughness height. Their transient-growth-based transition relations agree with the trend of Reda (1981, 2002) and the passive nose-tip technology data. However, the exact receptivity mechanisms

**Figure 7**

(a) A schematic of the actuator locations and (b) pressure perturbations at the same frequency for seven cases of unsteady simulations, where $s$ is the natural coordinate along the wedge. Figure taken from Wang et al. (2011c) with permission of the American Institute of Aeronautics and Astronautics.

**Figure 8**

Contour of pressure fluctuations induced by a streamwise velocity perturbation: (a) from the leading edge to approximately 0.45 m and (b) further downstream. Figure reprinted with permission from Wang & Zhong (2009). Copyright 2009, American Institute of Physics.
of the optimal transient growth associated with stationary streamwise vortices induced by surface roughness, which are critical to the explanation of transition, are still not known.

For hypersonic boundary layers, there has been a strong recent interest in transition affected by surface roughness. Iyer et al. (2011) investigated the effects of discrete and distributed roughness by DNS based on an unstructured grid. The test cases were an isolated cylindrical trip on a Mach-8.12 boundary layer, an isolated hemispherical bump on Mach-3.37, -5.26, and -8.23 boundary layers, and distributed roughness on a Mach-2.9 boundary layer. Their results showed that the roughness has much more difficulty tripping the boundary layer at higher Mach numbers. Figure 9 shows the isocontour of the Q criteria plotted with the streamwise velocity for the Mach-3.37 case. Coherent streamwise vortices are produced, which break down far downstream. Prominent hairpin-shaped structures and wall-normal and spanwise inflection points in streamwise velocity are observed as flow transition occurs.

Bartkowicz et al. (2010a) performed DNS studies of Mach-6 flow around an isolated roughness element, related to experiments conducted at the Purdue Mach-6 Quiet Tunnel (Wheaton & Schneider 2010). Three test cases corresponding to Reynolds numbers in the range of 45,000–55,000 were considered, in which transition occurred in the highest–Reynolds-number case. Chang et al. (2010) computationally investigated unsteady wake development behind large isolated cylindrical roughness elements. Their results showed that the wake is characterized by a mushroom-shaped streak and horseshoe vortices. The oscillatory vortices eventually lead to early vortex breakdown for the largest roughness element. Marxen et al. (2010) conducted a DNS study of the disturbance amplification in a Mach-4.8 flat-plate boundary layer with a localized two-dimensional roughness element. The small disturbances at a fixed frequency were introduced by
Pressure perturbations induced by imposed mode S and surface roughness. Figure taken with permission from Duan et al. (2010).

Wall blowing and suction upstream of the roughness element. They found that the roughness element considerably alters the instability of the boundary layer, leading to increased amplification or damping of a modal wave depending on the frequency range. Balakumar (2008) studied the receptivity to roughness and free-stream sound for supersonic flows over axisymmetric cones. Redford et al. (2010) used DNS to study flow over a roughness element in a flat-plate high-speed boundary layer. The roughness element is modeled as a continuous bump with a height approximately half of the boundary-layer thickness.

Wang & Zhong (2008) studied the receptivity of a Mach-5.92 flow over a flat plate to small three-dimensional surface roughness. They found that there is a pair of counter-rotating streamwise vortices induced by surface roughness, and the energy norm only has a small growth in the wake. Recently, Duan et al. (2010) used their high-order cut-cell method to simulate a Mach-5.92 flat-plate boundary layer under the combined effect of arbitrary finite-height surface roughness and wall blowing and suction. The height of the roughness element is half of the local boundary-layer thickness. Figure 10 shows the contours of pressure perturbations induced by imposed mode S and surface roughness. The roughness element is located on the surface at approximately $x = 0.185$ m. The pressure perturbations are reduced significantly after passing the roughness element. Moreover, when the roughness element is located upstream of the synchronization point, the instability wave develops in the same way as the no-roughness case. The unstable second mode is destabilized when the roughness element approaches the synchronization point. Conversely, when the roughness is located around and downstream of the synchronization point, the roughness element stabilizes the unstable second mode. Duan & Zhong (2010) also applied their method to a three-dimensional Mach-5.92 flow over a flat plate with an array of roughness elements.
5.4. Real Gas Effects

Practical hypersonic flow involves high-enthalpy free-stream conditions leading to high gas temperatures in the boundary layer behind the bow shock. As a result, real gas effects, which include vibrational excitation, species dissociation and recombination, ionization, and radiation (Bertin & Cummings 2006), become significant. In addition, the ablation of surface ablative thermal protection systems can have strong effects on the flow. Therefore, a study of the hypersonic boundary-layer transition cannot neglect real gas effects and surface chemistry.

There have been a limited number of experimental studies on the real gas effects on the transition of high-enthalpy hypersonic boundary layers (Hornung et al. 2002). Most theoretical studies of the real gas effects on the boundary-layer transition have been carried out using LST or PSEs. Examples of these studies include Malik & Anderson (1991), Stuckert & Reed (1994), Hudson et al. (1997), Johnson et al. (1998), Malik (2003), and Ulker et al. (2011). Bertolotti (1998) used linearized Navier-Stokes equations with nonequilibrium models to study the influence of rotation-vibration energy relaxation on boundary-layer stability. Stuckert & Reed (1994) showed that real gas effects destabilize the second mode while stabilizing the first mode. Johnson et al. (1998) conducted $e^N$ calculations on the effects of free-stream enthalpy on the hypersonic boundary-layer transition. They found that there is an increase in the transition Reynolds number with increasing free-stream total enthalpy, which is consistent with the experimental findings of Hornung et al. (2002).

DNS studies on the stability and receptivity of hypersonic nonequilibrium flows are currently in an initial stage. Ma & Zhong (2004) studied the receptivity to free-stream disturbances of a Mach-10 oxygen flow over a flat plate. They found that real gas effects destabilize the second mode. Some initial results were reported by Parsons et al. (2010) on the receptivity to free-stream disturbances of a Mach-15.3 flow with thermochemical nonequilibrium over a blunt cone. Stemmer (2006) conducted DNS studies on the influence of real gas effects on hypersonic flat-plate boundary-layer instability. A Mach-20 flow over a flat plate was considered with five-species air models including nonequilibrium effects. Their results showed a decrease in disturbance levels for three-dimensional waves, whereas the two-dimensional second mode is almost unchanged. Linn & Kloker (2010) investigated the effect of thermochemical nonequilibrium on the hypersonic boundary-layer transition by DNS. They found a stabilizing effect on disturbance waves in nonequilibrium gas.

6. TRANSITION

6.1. Transitional High-Speed Boundary Layers

When the boundary-layer waves reach a certain amplitude, the nonlinear secondary instabilities and three-dimensional effects lead to breakdown to turbulence with the appearance of turbulent spots in transitional boundary layers. The initial generation and development of turbulent spots in supersonic and hypersonic boundary layers is important for the understanding of the natural transition scenario in which a broad range of disturbances can be excited by a delta function such as a point source. This complex nonlinear interaction between a wide range of waves can be captured by DNS studies. Most of earlier DNS studies of turbulent spots were for incompressible flows.

DNS of the growth of a turbulent spot in supersonic boundary layers at Mach 2, 4, and 6 was performed by Krishnan & Sandham (2006). The forcing amplitudes are large enough so that the boundary layer undergoes a bypass transition. Their results showed the typical structures seen in the incompressible case. In the Mach-6 case, they found evidence of a supersonic (Mack) mode
substructure and coherent spanwise structures under the spot overhang region. Jocksch & Kleiser (2008) performed SDNS to investigate the growth of isolated turbulent spot in laminar zero-pressure-gradient flat-plate boundary layers at Mach 1.1 and 5. Figure 11 shows the simulated turbulent spots for the Mach-5 case.

More recently, Mayer et al. (2011a) and Sivasubramanian & Fasel (2010, 2011) conducted a series of SDNS studies on the transition initiated by a localized disturbance in supersonic and hypersonic boundary layers over a flat plate and a circular sharp cone. For the case of Mach-6 flow over a sharp cone, the base flow was obtained by a separate simulation using a high-order finite-volume shock-capturing scheme. The boundary layer was forced by a short duration pulse through a hole on the surface to model a natural transition scenario. The pulse developed into a three-dimensional wave packet with a wide range of frequencies and wave numbers. For a low-amplitude pulse, the dominant waves within the wave packet were identified as two-dimensional second-mode waves. The response of the flow to a high-amplitude pulse indicated the presence of a fundamental resonance mechanism. However, lower-amplitude secondary peaks were also identified in the wave spectrum at approximately half the peak frequency, an indication of a subharmonic resonance mechanism. Figure 12 shows a snapshot of the nonlinear wave packet. Three-dimensional (oblique) wave fronts can be seen along the lateral sides of the wave packet.

Figure 11
Snapshot of isosurfaces of $\lambda_2 = 10^{-5}$ colored with velocity magnitude: bottom view (top panel), side view (middle panel), and top view (bottom panel). Figure reprinted from Jocksch & Kleiser (2008), copyright © 2008, with permission from Elsevier.
6.2. Nonlinear Breakdown of High-Speed Boundary Layers

For incompressible boundary layers, secondary instabilities involving fundamental and subharmonic resonances were identified as two relevant mechanisms of boundary-layer transition (Kachanov 1994). The physical mechanisms of nonlinear breakdown for supersonic and hypersonic boundary layers are much more complex than their incompressible counterpart. Experimental results that can be used to study the nonlinear flow physics are limited for supersonic boundary layers (Kosinov et al. 1990). So far there is no general agreement on the dominant mechanisms for the breakdown in the hypersonic regime. For hypersonic boundary layers, although fundamental and subharmonic breakdown mechanisms are both possible, DNS studies have found that they may not be dominant mechanisms because first-mode instabilities are most unstable when they are oblique. Based on DNS results, Thumm et al. (1990) discovered a new oblique breakdown mechanism that is caused by a pair of oblique waves at the same frequency with identical but opposite spanwise wave numbers. They showed that, in a Mach-1.6 boundary layer, the transition is induced by the pair of oblique waves. Chang & Malik (1994) used PSEs to analyze the same Mach-1.6 boundary layer and showed that the dominant mechanism for transition is associated with the breakdown of oblique first-mode waves.

The oblique breakdown and the secondary-instability breakdown mechanisms have been extensively studied by TDNS in the early 1990s and by SDNS more recently. Pruett & Zang (1992) conducted a TDNS study on the subharmonic nonlinear breakdown of a Mach-6.8 boundary layer over a sharp cone. More TDNS studies were completed by several groups, including Dinavahi et al. (1994) on fundamental and subharmonic resonances for a Mach-4.5 boundary layer, Sandham et al. (1995) for the oblique breakdown of a Mach-2 boundary layer, Guo et al. (1995) for the nonparallel effects on oblique breakdown of the boundary layer initially considered by Thumm et al. (1990), and Adams & Kleiser (1996) on the subharmonic transition of a Mach-4.5 flat-plate boundary layer induced by a second-mode instability with low-amplitude random noise.

With some exceptions (Guo et al. 1996), TDNS does not account for nonparallel effects and is not as practical as SDNS. Rai & Moin (1993) developed high-order-accurate, upwind-biased algorithms for the SDNS of low-speed compressible boundary layers. They demonstrated the essential feature of the transition process as a result of free-stream broadband disturbances for a compressible flat-plate boundary layer. Maestrello et al. (1991) studied the nonlinear interaction between the first- and second-mode waves in a Mach-4.5 flat-plate boundary layer. It was

Figure 12
Snapshot of a nonlinear wave packet developing in the downstream direction. Figure taken with permission from Sivasubramanian & Fasel (2011).
found that the two-dimensional second mode causes a significant increase in the nonlinearity and three-dimensionality of the flow field. Bestek et al. (1993) conducted SDNS investigations on a Mach-1.5 flat-plate boundary layer for the oblique breakdown to turbulence, initiated by a pair of oblique disturbance waves. Instead of staggered or aligned lambda vortices arising from secondary instability, they found the development of a honeycomb-like structure in the breakdown (Figure 13). Guo et al. (1994) reviewed TDNS and SDNS studies of the compressible boundary-layer transition for flat-plate boundary layers at Mach numbers from 1.6 to 4.5. Both the subharmonic transition and oblique breakdown were investigated. Eibler & Bestek (1996) studied the spatial development of disturbances with small and moderate amplitudes in a two-dimensional Mach-4.8 flat-plate boundary layer. In simulations for larger-amplitude wall blowing and suction, fundamental resonance is observed, in which both types of three-dimensional waves are nonlinearly amplified and synchronize with the two-dimensional disturbances. Subharmonic resonance is also found for three-dimensional waves with large wave numbers.

Pruett and his colleagues at NASA have conducted extensive SDNS studies of the hypersonic boundary-layer transition for a Mach-4.5 flow over a flat plate (Pruett et al. 1995), a Mach-8 flow over a sharp cone with a half-angle of 7° (Pruett & Chang 1995, Stetson et al. 1983), and a Mach-6 flow over a sharp flared cone (Pruett & Chang 1998). Their SDNS results were verified by a comparison with the corresponding PSE results (Pruett et al. 1995). For the Mach-8 flow over a sharp cone, the transition is triggered by a symmetric pair of oblique second-mode disturbances. The nonlinear interactions of oblique waves generate strong streamwise vorticity and severe spanwise variations in the flow, which lead to eventual breakdown. Similar SDNS were also applied to the Mach-6 flow over an axisymmetric flared cone in which the transition was induced by periodic forcing derived from PSE calculations. They found significant qualitative differences between the flared-cone results and those obtained for a straight cone.

More recently, Fezer & Kloker (1999) conducted DNS studies of nonlinear breakdown for Mach-2 and -6.8 flat-plate boundary layers. They found that oblique breakdown is likely dominant as compared with the subharmonic resonance type of breakdown. Jiang et al. (2006) performed
DNS of a Mach-4.5 flat-plate boundary layer undergoing the oblique breakdown to a fully turbulent state. Muppidi & Mahesh (2010) performed DNS of the transition induced by wall blowing and suction in a spatially evolving Mach-2.25 boundary layer. Li et al. (2010) performed DNS of the transition for a Mach-6 flow over a blunt cone. Lu et al. (2011) used DNS results to study transition mechanisms for compressible boundary layers at subsonic speeds. Fasel and his group have conducted extensive DNS studies of both incompressible and compressible boundary layers (Fasel 2006). Recently, they performed a series of SDNS studies on the nonlinear breakdown of high-speed boundary layers (a) to seek experimental evidence of oblique breakdown, which was discovered by DNS (Thumm et al. 1990), and (b) to focus on the relationship between oblique breakdown and asymmetric subharmonic resonance. An example of these studies includes the DNS study of Mayer et al. (2011c) on the transition process in a Mach-2 flat-plate boundary layer to a point source, corresponding to the experimental investigations by Kosinov et al. (1994). Mayer et al. (2011b) also performed the DNS of the complete transition to turbulence via oblique breakdown in a Mach-3 flow. The results demonstrate that oblique breakdown can lead to a fully developed turbulent boundary layer. Similar DNS studies were also extended to supersonic and hypersonic flows over sharp and blunt cones (Husmeier & Fasel 2007, Koevary et al. 2010, Laible et al. 2009).

7. THREE-DIMENSIONAL AND COMPLEX BOUNDARY LAYERS

Most DNS studies on hypersonic receptivity, instability, and transition have been conducted for simple two-dimensional or axisymmetric base flows, such as hypersonic flows over flat plates with sharp or blunt leading edges, straight or flared sharp cones, blunt cones with spherical noses, and straight or curved sharp or blunt wedges at zero angle of attack. However, practical hypersonic vehicles have complex three-dimensional geometry. DNS studies in three-dimensional hypersonic boundary layers have just started in recent years.

An interesting three-dimensional hypersonic boundary layer is flow over cones with elliptic cross sections, which have similar geometry to hypersonic lifting vehicles (Kimmel et al. 1999). Cross-flow instability plays an important role in the boundary-layer transition of such a flow. Zhong (1999) performed a DNS study on the receptivity of a three-dimensional Mach-15 boundary layer to free-stream acoustic disturbances for a blunt 2:1 elliptic cone. More recently, Bartkowicz et al. (2010b) presented DNS results of the transition in a Mach-8 boundary layer over a sharp 4:1 elliptic cone. Acoustic noise was added into the free stream to model the experimental conditions. The interaction between the cross-flow instability and center-line waves was investigated. Figure 14 shows the streamwise cross-flow vortices starting at a point at a distance from the leading edge and then continuing down the length of the cone.

Here we briefly mention other DNS studies on three-dimensional boundary-layer stability and transition. Balakumar & Owens (2010) performed a series of DNS studies on the stability and receptivity to free-stream disturbances for hypersonic and supersonic boundary layers over cones at an angle of attack, over swept wings (Balakumar & King 2010), and over a compression corner (Balakumar et al. 2005). Jiang et al. (2004) performed DNS of cross-flow disturbances in supersonic boundary layers. Li et al. (2010) studied nonlinear breakdown of a Mach-6 boundary layer over a cone at a small angle of attack of 1°. Random wall blowing and suction were used to trigger the transition. Pagella et al. (2002) conducted numerical investigations of small-amplitude disturbances in a Mach-4.8 boundary layer with an impinging shock wave. Whang & Zhong (2003) studied the leading-edge receptivity of Görtler vortices for a Mach-15 flow over a blunt concave wedge. They showed that Görtler vortices are mainly induced by streamwise vorticity.
waves. Sesterhenn & Friedrich (2006) numerically studied the effects of compressibility and nose radius on instabilities near the attachment line of swept wings.

8. NOSE-BLUNTNESS/ENTROPY-LAYER EFFECTS AND TRANSITION REVERSAL

The bow shock in front of a blunt nose has strong effects on the stability and transition of boundary layers behind it (Morkovin 1987, Reshotko 1991). The effects of nose bluntness and the entropy layer on the hypersonic boundary-layer transition have been studied in experiments (Stetson et al. 1984). Experimental results showed a transition reversal: The increase in nose bluntness delays transition, but the trend is reversed when the nose bluntness exceeds a certain limit, as shown in Figure 15. The stability characteristics of hypersonic boundary layers over a blunt cone corresponding to Stetson’s experiments have been studied by many researchers using linear stability analysis (Kufner & Dallmann 1996, Malik et al. 1990). Other LST studies on blunt cones in a hypersonic free stream have been conducted by Dietz & Hein (1999), Su & Zhou (2007), and Heffner & Arnal (1994). Linear stability results do not show the reversal of instability due to nose bluntness. These previous stability analyses, however, were conducted mainly on test cases in which the actual transition reversal was not observed in experiments. Lei & Zhong (2010) conducted a linear stability analysis on Stetson & Rushton’s (1967) Mach-5.5 experiments in which the reversal was observed. Related DNS studies were carried out by Zhong (2009, 2011). He used three different nose radii of 0.156, 0.5, and 1.5 inches, covering both the small and large bluntness regions. It was found if only the second-mode instabilities are considered, the transition will always be delayed as the nose bluntness increases, contrary to experimental observation.

With regard to the entropy layer and bow-shock effects on the boundary-layer transition, DNS is an ideal tool because the boundary layers with a bow shock can be simulated without much simplification. Zhong & Ma (2006) performed DNS studies of Stetson’s blunt cone in Mach-8 flows by using their high-order shock-fitting codes. The numerical instability growth
compared well with experimental results. Zhong (2005) also studied the effects of nose bluntness on the receptivity to free-stream acoustic waves for Stetson’s blunt cone. No reversal of receptivity was found. Kara et al. (2007) performed a similar DNS study on the effects of nose bluntness on the stability of a Mach-6 boundary layer over a 5° straight cone. They found that the bluntness has a strong stabilizing effect on the boundary layers. Husmeier & Fasel (2007) investigated the bluntness effects on nonlinear breakdown. Again, they found only stabilization effects on the nose bluntness.

So far the issue of transition reversal is still not resolved. There may be a possibility that the reversal is a consequence of tunnel noise in the experiments in which the reversal was observed. Further DNS and theoretical studies are expected in this area. The quiet tunnel technology is also valuable to resolve this issue.

9. OTHERS

Other related issues are also important for the DNS studies of the hypersonic boundary-layer transition, such as the theoretical interpretation of DNS results and transition control. Because these topics are not related to the DNS of the physics of the transition mechanisms, they are discussed briefly in this section.

9.1. Theoretical Analysis of Direct Numerical Simulation Results

The DNS of hypersonic boundary-layer receptivity, instability, and transition provides a vast amount of information about the transient flow field, requiring theoretical analysis to understand and interpret the physical mechanisms underlining the results. Recently, the biorthogonal multimode decomposition was used to analyze perturbations in compressible and incompressible boundary layers (Tumin 2003). Tumin et al. (2007, 2011) applied the multimode decomposition.
to analyze DNS results of hypersonic boundary layers over a flat plate (Tumin et al. 2011) and a sharp wedge (Wang et al. 2011c). Figure 16 compares the theoretically predicted receptivity coefficient with that filtered out from the computational results for the case of a Mach-8 flow over a sharp wedge. The figure shows that there is good agreement between receptivity coefficients calculated with the help of the receptivity model and those obtained from the numerical results as a projection onto the normal modes. For the Mach-5.92 flat-plate boundary layer, Tumin et al. (2011) decomposed the perturbation flow field into normal modes with the help of the multimode decomposition. Although the DNS data for the wall pressure perturbation have wiggles near the actuator region due to the coexistence of various modes, the filtered-out amplitude of the unstable mode S is smooth, and it is in good agreement with the theoretical prediction. The filtered-out decaying mode F is also in good agreement with the theoretical prediction.

The theoretical analysis of DNS results is not only useful for verification but also for the interpretation of physical mechanisms hidden behind the results. So far theoretical analyses of DNS results, such as the analysis using the multimode decomposition, have not been adopted widely. Most current DNS results on hypersonic boundary-layer stability and transition are still evaluated with LST in which only DNS results far downstream of an actuator can be compared with theoretical prediction.

9.2. Transition Control Using Surface Porous Coatings

Passive control of the hypersonic boundary-layer transition using porous coatings has been successfully demonstrated by theoretical analyses (Fedorov et al. 2001), experiments (Fedorov et al. 2003, Rasheed et al. 2002), and numerical simulations (Egorov et al. 2008, Wang & Zhong 2010). Fedorov et al. (2001) performed theoretical analyses on the second-mode instability of a hypersonic boundary layer over a flat plate covered by an ultrasonically absorptive coating. They found that the second-mode growth is reduced because the porous layer absorbs the disturbance energy. Both theoretical predictions and experimental measurements showed that the porous coating stabilizes the second mode and marginally destabilizes the first mode. Stephen & Michael (2010) theoretically considered the effect of a porous wall on Mack's first mode of a hypersonic boundary layer on a sharp slender cone. They found that the porous wall significantly destabilized the non-axisymmetric modes.
Egorov et al. (2008) studied the effect of porous coatings on the stability and receptivity of a Mach-6 flat-plate boundary layer by two-dimensional numerical simulations. They found that a regular-structure porous coating effectively diminishes the second-mode growth rate, while weakly affecting acoustic waves. In previous studies, porous coatings cover either the entire flat plate or the surface around half the cone circumference. Wang & Zhong (2010) conducted numerical studies of the stabilization of a Mach-5.92 flat-plate boundary layer by putting local sections of felt-metal porous coatings both upstream and downstream of the synchronization point. Disturbances corresponding to mode S or mode F are superimposed at a cross section of the boundary layer near the leading edge. Parameters of the felt-metal porous coating are the same as those used by Fedorov et al. (2003). For the case of mode S, the porous coating destabilizes the first mode, whereas it stabilizes the second mode. In addition, the destabilization of the first mode could be significant for the felt-metal porous coating. The results showed that the porous coating generally stabilizes mode F in porous regions. In a numerical simulation to study the effect of different porous coatings, Wang & Zhong (2011b) also investigated mode-S growth over felt-metal and regular porous coatings. The results showed that at approximately the same porosity, the regular porous coating is weaker in first-mode destabilization and second-mode stabilization.

10. SUMMARY AND FUTURE DIRECTIONS

Above we review DNS studies on the receptivity, instability, and transition of hypersonic boundary layers in the past 20 years. Significant progress has been made on DNS in the area of hypersonic flow and in the use of SDNS. Various high-order shock-capturing and shock-fitting finite-difference methods have been developed and extensively applied to the DNS of the hypersonic boundary-layer transition. Theoretical tools, including multimode decomposition and transient growth theory, have been used to analyze DNS data and study the physical mechanisms hidden behind the data. With the rapid improvement of efficient parallelizable numerical algorithms and high-performance computers, DNS has now become a powerful and mature research tool for the study of the hypersonic boundary-layer transition and has led to new discoveries in transition mechanisms.

In the past 20 years, DNS studies on hypersonic boundary-layer receptivity and instability have shown that the main receptivity mechanisms of hypersonic boundary layers are different from those of low-speed boundary layers. Resonant interactions between forcing waves and boundary-layer waves are the main receptivity mechanisms in hypersonic boundary layers. For high-enthalpy hypersonic flow, real-gas effects are found to stabilize boundary-layer flows. Nonlinear breakdown mechanisms of high-speed boundary layers, including fundamental, subharmonic, and oblique breakdown, have been studied with the help of DNS tools. DNS studies on three-dimensional boundary layers and nose-bluntness/entropy-layer effects have begun in recent years.

Despite considerable efforts in experimental, theoretical, and numerical studies, many critical physical mechanisms underlying the hypersonic boundary-layer transition are still poorly understood. It is expected that current DNS studies on the receptivity, instability, and transition of hypersonic boundary layers will lead to a better understanding of transition mechanisms in the high-speed regime. Several problems are expected to be studied extensively in the near future, including the receptivity of hypersonic boundary layers to various disturbances, nonlinear breakdown mechanisms, three-dimensional and complex boundary layers, transient growth at hypersonic speeds, roughness-induced transition, and the effects of nose bluntness and the entropy layer on transition and transition-reversal phenomena. Although there are some experimental
and numerical studies on the transition of hypersonic boundary layers including real gas effects, DNS on the hypersonic boundary-layer transition with real gas effects and with surface ablation is currently in an initial stage. It is anticipated that major progress will also be made in this area. Meanwhile, the theoretical interpretation of DNS results is also important. Extensive theoretical analyses are expected on DNS data by using multimodal decomposition and transient growth theory. The passive control of the hypersonic boundary-layer transition by using surface porous coatings has motivated more and more research, and it is on the way toward practical application.

The ultimate goal of DNS of the hypersonic boundary-layer transition is the ability to simulate the complete transition process in practical vehicle geometry and in a realistic forcing environment. Currently, such a task is still beyond the computational power of today’s supercomputers. However, with the coming of petascale supercomputers and the intelligent incorporation of results of receptivity studies into a DNS simulation, DNS on the complete process of the hypersonic boundary-layer transition, from receptivity to nonlinear breakdown, has the potential of becoming a reality in the near future.

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The authors are not aware of any biases that might be perceived as affecting the objectivity of this review.

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LITERATURE CITED
Balakumar P. 2009. Receptivity of a supersonic boundary layer to acoustic disturbances. AIAA J. 47:1069–78


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Stetson KF, Rushton GH. 1967. Shock tunnel investigation of boundary-layer transition at $M = 5.5$. AIAA J. 5:899–906


